P425/1
PURE MATHEMATICS
Paper 1
25 July 2018
3 hours

ENTEBBE JOINT EXAMINATION BUREAU

Uganda Advanced Certificate of Education

MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Attempt **ALL** *the eight questions in* Section **A** *and any* **five** *from* Section **B.**

Begin every answer on a fresh page.

Any additional questions answered will **not** *be marked.*

Mathematical tables and squared paper shall be provided

Silent, non – programmable calculators may be used.

State the degree of accuracy at the end of each answer attempted using a calculator or table and indicate **cal** *for calculator or* **tab** *for mathematical table.*

SECTION A: 40 MARKS

Attempt **all** *questions in this* Section.

1. Solve the equation $1 - 2\sin\theta - 4\cos2\theta = 0$ for values of 0 between 0^0 and 360^0. *(05 marks)*

2. If $(x-2)^2$ is a factor of $x^3 + ax^2 + bx + 12$. Find the values of a and b. *(05 marks)*

3. Find the distance from the centre of the circle $x^2 + y^2 - 4x + 2y - 2 = 0$ to the line $3x - 4y + 5 = 0$. *(05 marks)*

4. Differentiate $\sin^2 x$ from first principles. *(05 marks)*

5. Find the possible values of k if the quadratic equation $2kx^2 - 8x + 1 = 2k(x-2)$ has equal roots. *(05 marks)*

6. Use Maclaurin's theorem to find the first 3 non zero terms of the expansion $\ln(1-2x)$. *(05 marks)*

7. Find the coordinates of the point of intersection of the lines

$$r = \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{ and } r = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

(05 marks)

8. Find $\displaystyle\int \frac{2x^2}{\cos^2(x^3+2)}\,dx$ *(05 marks)*

SECTION B

9. (a) Given that $z_1 = 2$ and $z_2 = 3 + 2\boldsymbol{i}$. find the argument of

$$\left(\frac{z_1 + z_2}{z_2 - z_1}\right)$$

(06 marks)

(b) Given that $z = -3$ is a root of the equation $z^3 - 5z^2 + az + 60 = 0$. Find the value of a and hence find other roots. *(06 marks)*

10. (a) Find the mean value of the function $y = \sqrt{x}$ from $x = 0$ to $x = 4$.
(05 marks)

(b) Given that y = $\ln \sin t$ and $x = 1 - \cos t$. find in $\frac{d^2y}{dx^2}$ terms of t.
(07 marks)

11. (a) Show that $\frac{\cos(2x + 60^0) - \cos(2x - 60^0)}{\cos(x + 30^0) - \cos(x - 30^0)} = 2\sqrt{3}\cos x$ *(05 marks)*

(b) Given that $\tan A = \frac{4}{3}$ and $\tan (A + B)$ $\frac{63}{16}$ Find the value of $\sin(A - B)$ without using tables or calculators if A and B are acute angles. *(07 marks)*

12. (a) Expand $(1 + 2x)^{½}$ in ascending powers of x up to the term in x^3. By putting $x = \frac{1}{25}$ find an approximate value of $\sqrt{3}$ to 3 decimal places.
(06 marks)

(b) Given that $(1 + 3x)^n = 1 + 15x + 6mx^2 + \ldots$. Find the values of constants m and n. *(06 marks)*

13. (a) If vectors $3\boldsymbol{i} + a\boldsymbol{j} + 2\boldsymbol{k}$ and $-6\boldsymbol{i} + a\boldsymbol{j} + \boldsymbol{k}$ are perpendicular, find the value of a given that a < 0. *(03 marks)*

(b) Find the vector equation of a line through the point $P (6, -1, -5)$ which is perpendicular to the plane $x + 2y - 2z = 5$.

Hence obtain the coordinates of the point of intersection and deduce the distance from point P to the above plane. *(09 marks)*

14. The gradient of a curve at the point p (x, y) is $\frac{x^2 + 1}{y}$ The point A (3, 12) lies on the curve. Find:

(i) equation of the normal to the curve at A. *(05 marks)*

(ii) equation of the curve and the point where it cuts the y – axis.
(07 marks)

Turn Over

15. (a) If the line $y = 4x + c$ is a tangent to the parabola $y^2 = -16x$. Find the value of c. *(05 marks)*

(b) The line $y = ax - 1$ and the curve $y = x^2 + bx - 5$ intersect at ***P*** $(4, -5)$ and ***Q***. Find the values of a and b and the coordinates of ***Q***.

(07 marks)

16. (a) State the differential equation $y \frac{dy}{dx} = 2x(1 + y)$
Given that $y(0) = 2$. *(04 marks)*

(b) The rate at which malaria spreads in the body is proportional to the number of infected cells in the body. If the number of infected cells in the body at any time is N. given that after a month, the number of cells infected is doubled and considering the initial number of cells infected to be N_0.

(i) Show that $N = N_0 e^{t \ln z}$

(ii) Show that five months later the number of infected cells is $32\, N_0$.

(08 marks)

END